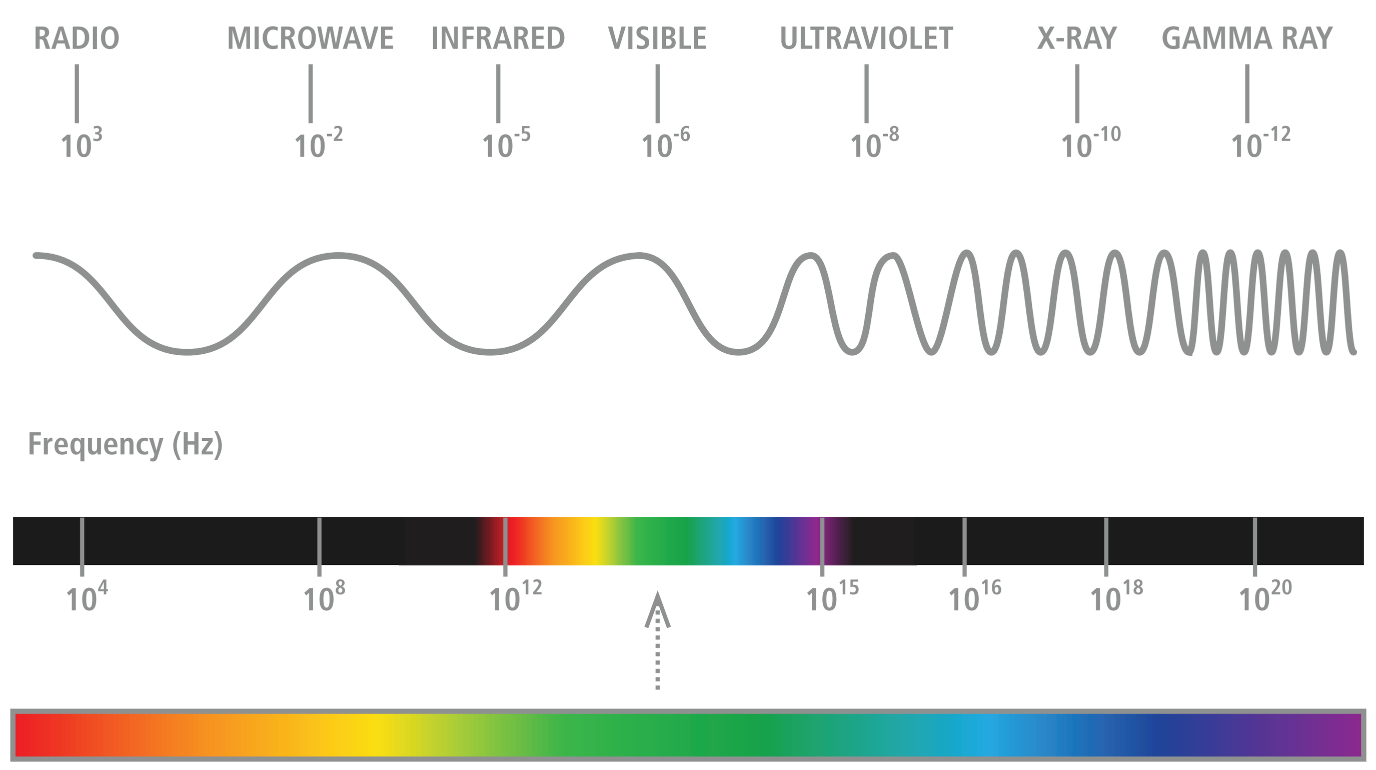
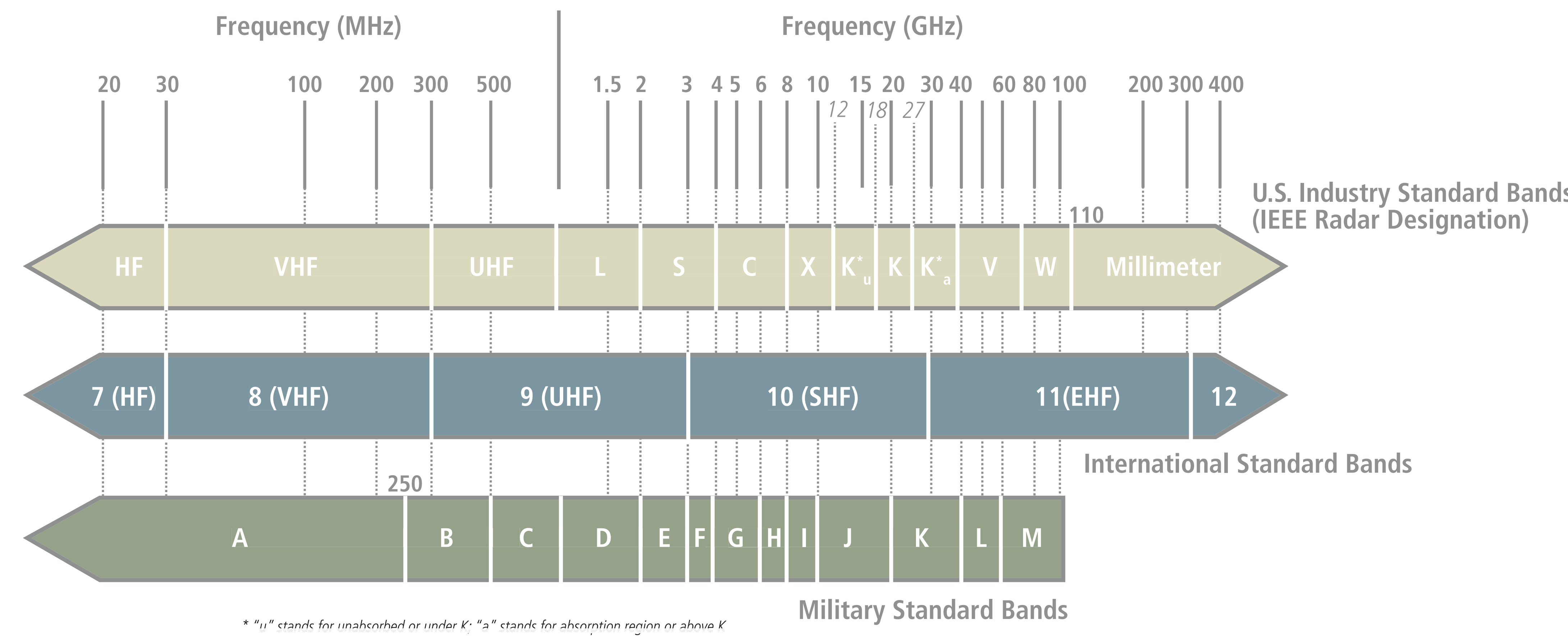
ELECTRONIC WARFARE QUICK REFERENCE GUIDE

THE ELECTROMAGNETIC SPECTRUM





Band Designation	Frequency Range
HF	3–30 MHz
VHF	30–300 MHz
UHF	300–1,000 MHz
L	1–2 GHz
S	2–4 GHz
C	4–8 GHz
X	8–12 GHz
K_u	12–18 GHz
K	18–27 GHz
K_a	27–40 GHz
V	40–75 GHz
W	75–110 GHz

RF Propagation — FRIIS TRANSMISSION EQUATION

$$P_r = P_t G_t G_r \left(\frac{\lambda}{4\pi R}\right)^2$$

Pr: Received Power
Pt: Transmit Power
Gt: Transmit Gain
Gr: Receive Gain
R: Range

RF Propagation — RADAR HORIZON

$$D_h = \sqrt{2HR_e}$$

H: Horizon
Re: Earth Radius ~ 6,371 km

RF Propagation — TARGET VISIBILTY

$$\text{Target Height} = \frac{(\text{Target Range} - \sqrt{2HRe})^2}{2Re}$$

H: Horizon
Re: Earth Radius ~ 6,371 km

Detection & Estimation Probability — MAX LIKELIHOOD ESTIMATION

Joint Density Function

$$f(x_1, x_2, ..., x_n|\theta) = f(x_1|\theta) \times f(x_2|\theta) \times ... \times f(x_n|\theta)$$

Likelihood

$$L(\theta; x_1, ..., x_n) = f(x_1, x_2, ..., x_n|\theta) = \prod_{i=1}^{n} f(x_i|\theta)$$

Log-Likelihood

$$\ln L(\theta; x_1, ..., x_n) = \sum_{i=1}^{n} \ln f(x_i|\theta)$$

Average Log-Likelihood

$$\hat{\ell} = \frac{1}{n} \ln L$$

$$\hat{\ell} = (\theta|x) = \frac{1}{n}\sum_{i=1}^{n} \ln f(x_i|\theta)$$

x_i: Observations
n: Number of Samples
f: Is one, or joint, probability distribution(s)
θ: Distribution parameters can be vectors

Electronic Warfare — NOISE JAMMING

$$S = \frac{\overbrace{P_{t_{radar}} G_{t_{radar}}}^{EIRP_{radar}} G_r \lambda^2}{(4\pi)^3 R^4} \sigma$$

$$J_{self} = \overbrace{P_{t_{jam}} G_{t_{jam}}}^{EIRP_{jam}} \left(\frac{\lambda}{4\pi R_{jr}}\right)^2 \frac{G_{r_{radar}}}{L_{r_{radar}}}$$

$$\frac{J}{S} = \left(\frac{EIRP_{jam}}{EIRP_{radar}}\right)\left(\frac{4\pi R^2}{\sigma}\right)$$

If $BW_{jam} \geq BW_{radar}$

$$\frac{J}{S} = \left(\frac{EIRP_{jam}}{EIRP_{radar}}\right)\left(\frac{4\pi R^2}{\sigma}\right)\left(\frac{BW_{radar}}{BW_{jam}}\right)$$

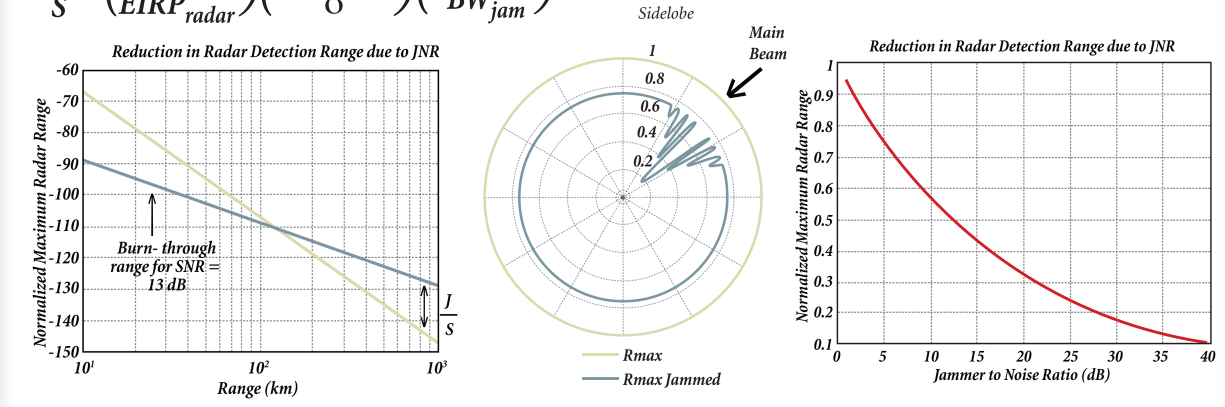
$$J/N \sim \left(\frac{R_{max}}{R_{max_{jammed}}}\right)^4$$

Assume: $J >> N$
$BW_{Jam} = BW_{Radar}$

Mainlobe

$$R_{max_{jammed}}{}^4 = \frac{P_t G'_t G'_r \lambda^2}{(4\pi)^3 (kT_s B_N N_f + J) * SNR * L_r * L_t} \sigma$$

Reduction in Normalized R_{max}



J_{self}: Self Protect Jammer Power
J/S: Jam to Signal Ratio at Radar Receiver
S: Radar Received Signal Power
$P_{t_{jam}}$: Jammer Transmit Power
$G_{t_{jam}}$: Jammer Transmit Gain
R_{jr}: Range between Jammer and Radar
R: Range between Radar Target and Radar
λ: Jammer Transmit Wavelength
$G_{r_{radar}}$: Radar Receiver Gain

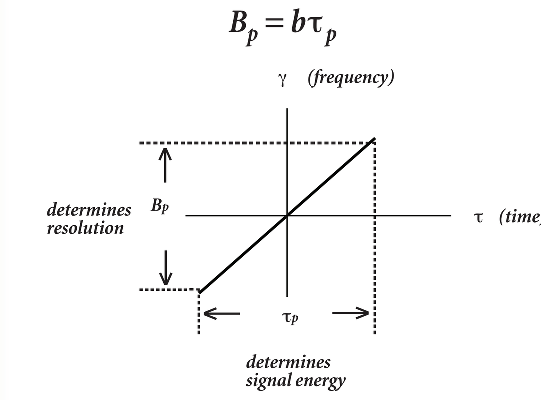
$L_{r_{radar}}$: Radar Receiver Losses
$P_{t_{radar}}$: Radar Transmit Power
$G_{t_{radar}}$: Radar Transmitter Gain
σ: Radar Target Radar Cross Section
BW_{Radar}: Radar Transmit Bandwidth
BW_{Jam}: Jammer Transmit Bandwidth
J: Jammer Power
$Rmax_{jammed}$: Jammed Radar Range
(Burn through Range)

R_{max}: Max Radar Range
J/N: Jammer to Noise Ratio
N: Total Noise
k: Boltzmann's constant
T_s: Receiver Temperature
B_N: Receiver Noise Bandwidth
SNR: Radar Signal to Noise Ratio
N_f: Receiver Noise Figure (>1)

Radar Processing — LINEAR FM WAVEFORM

$$s(\tau) = e^{j2\pi(f_c\tau + \frac{1}{2}b\tau^2)}, -\frac{\tau_p}{2} \leq \tau \leq \frac{\tau_p}{2}$$

$$B_p = b\tau_p$$



s(): Transmitted Signal Waveform
f_c: Center Frequency
τ: Range Time (fast time)
τ_p: Pulse Length
b: Chirp Rate
B_p: Pulse Bandwidth
γ: Range Frequency

Radar Processing — RADAR AMBIGUITY FUNCTION

$$x(\tau, t) = \int_{-\infty}^{\infty} s(t)s^*(t-\tau)e^{i2\pi ft}\, dt$$

S(t): Complex Baseband Pulse
τ: Time Delay
f: Doppler Shift

RF Propagation — WAVELENGTH

$$\lambda = \frac{c}{f}$$

Band	f	Wavelength
VHF	100 MHz	3.00 m
S	3 GHz	0.10m
C	6GHz	0.05m
X	10GHz	0.03m

c: Speed
f: Frequency

RF Propagation — DOPPLER SHIFT

$$f_d = -2v_r/\lambda$$

	X-band	S-band
Velocity	300 m/s	300 m/s
Wavelength	0.03 m	0.1 m
Doppler Shift	20 kHz	6 kHz

Detection & Estimation Probability — CRAMER RAO LOWER BOUND

$$CRB = \left(E\left\{\left[\frac{\partial \ln p(x,\theta)}{\partial\theta}\right]\left[\frac{\partial \ln p(x,\theta)}{\partial\theta}\right]^T\right\}\right)^{-1}$$

x: Observations
p: Probability distribution function (or joint)
θ: Distribution parameters can be vectors

Radar Processing — NOISE POWER

$$\text{Noise Power in Receiver} = kT_s B_N N_f$$

kT_s: = -174 dBm
K: Boltzmann's constant = $1.38*10^{-23}$ J/K
B_n: Noise Bandwidth
T_s: System Noise Temperature
T_s usually set to T_0 = 290K
N_f: Noise figure of receiver

Detection & Estimation Probability — BINOMIAL

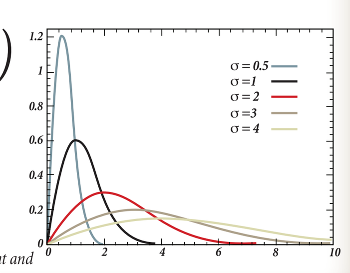
$$f(k; n, p) = Pr(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

p: Success probability of each trial
k: Number of successes
n: Number of trials

Detection & Estimation Probability — RAYLEIGH

$$p(r) = \begin{cases} \frac{r}{\sigma^2} e^{-\left(\frac{r^2}{2\sigma^2}\right)} \\ 0 \end{cases}$$

$(r < 0) \quad (0 \leq r \leq \infty)$



μ: Mean
σ: Standard Difference
A: Distance between the reference point and the center of the bivariate distribution

Fourier Relationships — PARSEVAL'S RELATION

$$\int_{-\infty}^{+\infty} |x(t)|^2\, dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |X(\omega)|^2\, d\omega$$

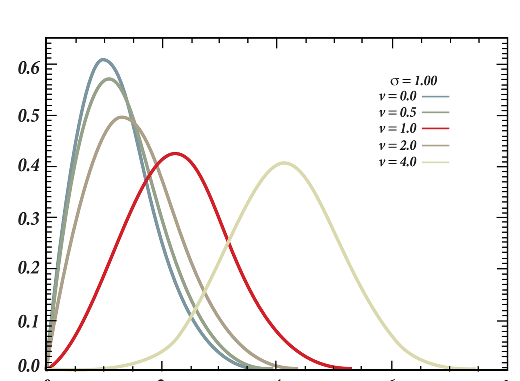
$$\frac{1}{T_o}\int_{T_o} |\tilde{x}(t)|^2\, dt = \sum_{k=-\infty}^{+\infty} |a_k|^2$$

Radar Processing — SPEED OF LIGHT

Speed of Light (approx)	Units
3x10^8	m/sec
300	m/usec
1.62x10^5	NM/sec
1x10^9	Ft/sec
1x10^3	Ft/usec

Detection & Estimation Probability — RICIAN

$$p(r) = \begin{cases} \frac{r}{\sigma^2} e^{-\frac{(r^2+A^2)}{2\sigma^2}} I_0\left(\frac{Ar}{\sigma^2}\right) & \text{for } (A \geq 0, r \geq 0) \\ 0 & \text{for } (r < 0) \end{cases}$$



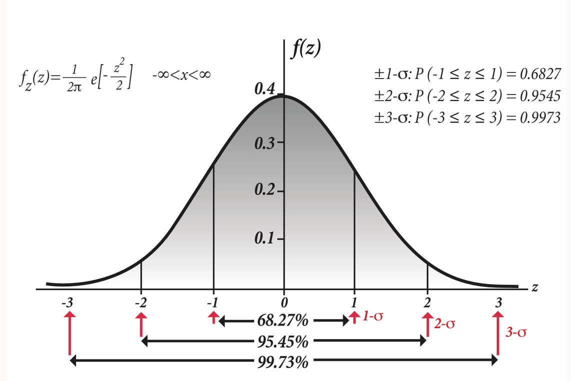
μ: Mean
σ: Standard Difference
A: Distance between the reference point and the center of the bivariate distribution
I_0: Bessel Function of the first kind with order zero

Detection & Estimation Probability — NORMAL

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)}{2\sigma^2}}$$

Standard Normal Curve

$(\mu_z = 0; \sigma_x = 1.0)$

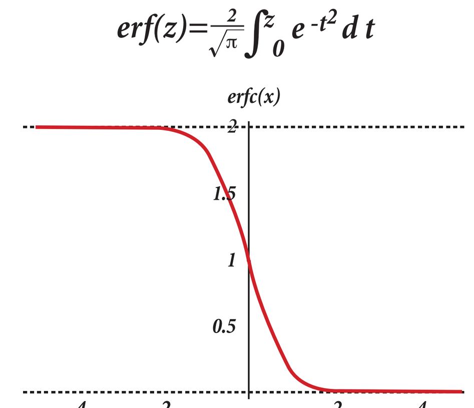


μ: Mean
σ: Standard Difference
A: Distance between the reference point and the center of the bivariate distribution

Detection & Estimation Probability — ERROR FUNCTIONS

$$erfc(z) = 1 - erf(z) = \frac{2}{\sqrt{\pi}}\int_z^{\infty} e^{-t^2}\, dt$$

$$erf(z) = \frac{2}{\sqrt{\pi}}\int_0^z e^{-t^2}\, dt$$



μ: Mean
σ: Standard Difference
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Fourier Relationships — CONTINUOUS-TIME FOURIER TRANSFORMATION

Synthesis

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(\omega)e^{j\omega t}\, d\omega$$

Analysis

$$X(\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t}\, dt$$

$$x(t) \overset{\mathcal{F}}{\leftrightarrow} X(\omega)$$

Fourier Relationships — MODULATION PROPERTY

Modulation

$$s(t)\,p(t) \overset{\mathcal{F}}{\leftrightarrow} \frac{1}{2\pi}[S(\omega)P(\omega)]$$

Convolution

$$h(t) * x(t) \overset{\mathcal{F}}{\leftrightarrow} H(\omega)X(\omega)$$

Time Shifting

$$x(t-t_0) \overset{\mathcal{F}}{\leftrightarrow} e^{-j\omega t_0} X(\omega)$$

Differentiation

$$\frac{dx(t)}{dt} \overset{\mathcal{F}}{\leftrightarrow} j\omega X(\omega)$$

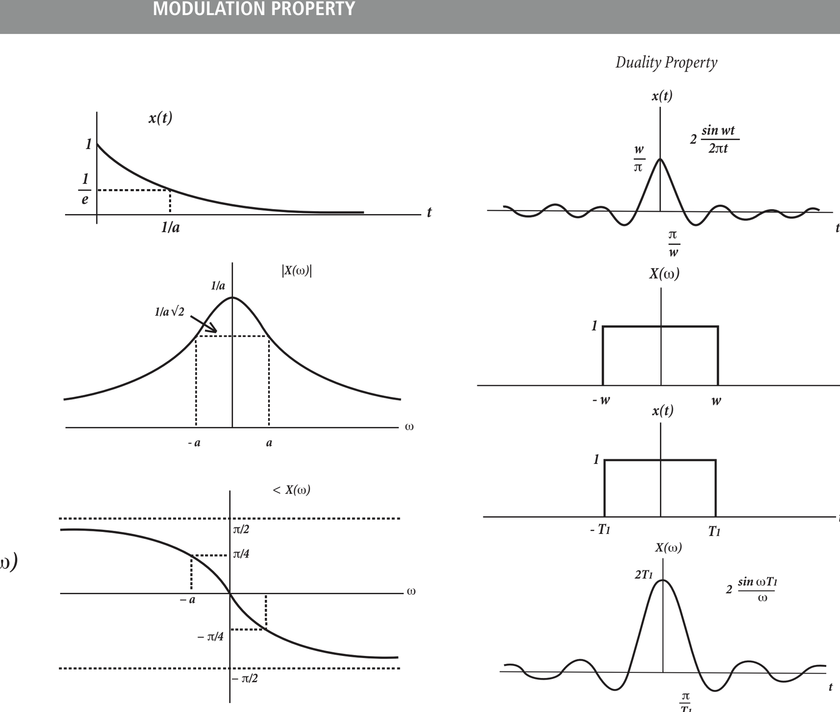
Integration

$$\int_{\infty}^{t} x(\tau)d\tau \overset{\mathcal{F}}{\leftrightarrow} \frac{1}{j\omega}X(\omega) + \pi X(0)\,\delta(\omega)$$

Linearity

$$ax_1(t) + bx_2(t) \overset{\mathcal{F}}{\leftrightarrow} aX_1(\omega) + bX_2(\omega)$$

Duality Property



Radar Processing — MAX UNAMBIGUOUS RANGE

$$R_{max} = \frac{c}{2PRF}$$

PRF	Range	Doppler
High	Ambiguous	Unambiguous
Medium	Ambiguous	Ambiguous
Low	Unambiguous	Ambiguous

PRF	Unambiguous Range
100 kHz	1.5 km
25 kHz	6 km
10 kHz	15 km

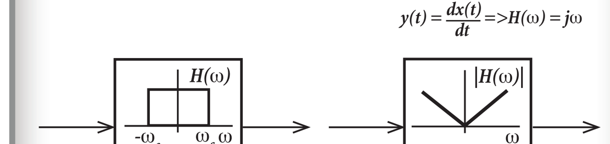
c: Speed of Light
PRF: Pulse Repetition Frequency

Fourier Relationships — FILTERING

Ideal Lowpass Filter

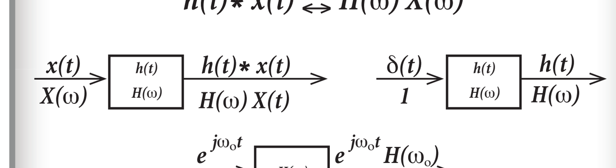
Differentiator

$$y(t) = \frac{dx(t)}{dt} \Rightarrow H(\omega) = j\omega$$



Convolution Property

$$h(t) * x(t) \overset{\mathcal{F}}{\leftrightarrow} H(\omega)\, X(\omega)$$



H(ω): Frequency Response
*: Convolution operation

Radar Processing — SIGNAL TO NOISE RATIO

$$SNR = \frac{P_R}{N_o} = \frac{P_t G_t G_r \sigma \lambda^2 G_p L}{(4\pi)^3 R^4 k_B T_s B_n N_f}$$

Pr: Received Power
Pt: Transmit Power
Gt: Transmit Gain
Gr: Receive Gain
R: Range
No: Noise Power
L: Losses

K: Boltzmann's constant = $1.38*10^{-23}$ J/K
B_n: Noise Bandwidth
T_s: System Noise Temperature
T_s usually set to T_0 = 290K
N_f: Noise figure of receiver

Antennas — ANTENNA BEAMWIDTH

Phased Array, Radians

$$\theta_{BW_{3dB}} \sim 0.886 \frac{\lambda}{Nd\cos\theta_0} b$$

Parabolic, Radians

$$\theta_{BW_{null}} \sim 1.22 \frac{\lambda}{d} \qquad \theta_{BW_{3dB}} \sim 0.88 \frac{\lambda}{d}$$

λ: Wavelength
d: Antenna Diameter

Antennas — ANTENNA DIRECTIVITY

$$D \approx 4\pi \frac{\left(\frac{180}{\pi}\right)^2}{\theta_{1d}\theta_{2d}} \approx \frac{40000}{\theta_{1d}\theta_{2d}}$$

θ_{1d}: Half-power beamwidth in one principal plane (degrees)
θ_{2d}: Half-power beamwidth in the other principal plane (degrees)

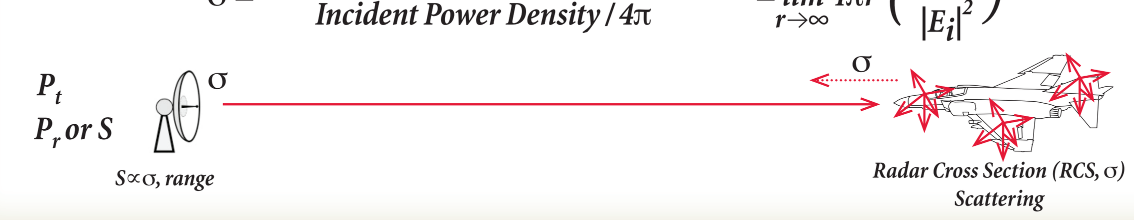
Antennas — ANTENNA GAIN

$$G_{ant} = \frac{4\pi A_e}{\lambda^2}$$

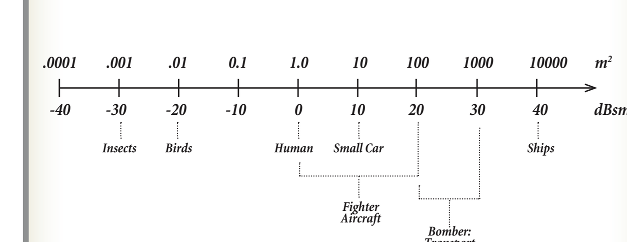
A_e: Effective Aperture Area
λ: Wavelength

Radar Processing — RADAR CROSS SECTION

$$\sigma = \frac{\text{Reflected Power to Receiver / Solid Angle}}{\text{Incident Power Density} / 4\pi} = \lim_{r\to\infty} 4\pi r^2 \left(\frac{|E_s|^2}{|E_i|^2}\right)$$



Radar Processing — TYPICAL VALUES OF RCS



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Raytheon